Homework #5 **Computational Fluid Dynamics I** Prof. V. Esfahanian
Due Date: 25/12/96 TA: A.Bahrami & M.Fazli

1. For $u_t = \alpha u_{xx}$ the following discretization is proposed

$$\frac{3}{2}\frac{\Delta_t u_i^n}{\Delta t} - \frac{1}{2}\frac{\nabla_t u_i^n}{\Delta t} = \frac{\alpha}{(\Delta x)^2}\delta_x^2 u_i^{n+1}$$

(a) write this out in suitable form for the computer using the variable $r = \alpha\Delta t/(\Delta x)^2$.
(b) Is this method explicit, or implicit? How many time levels are involved?
(c) Find the truncation error at the point (x_i, t_{n+1}).

2. Consider the following parabolic PDE:

$$u_t = \alpha u_{xx} - 2u \qquad 0 < x < 1, \qquad t > 0$$

with boundary conditions

$$\text{at} \quad x = 0 \qquad u = 1 \qquad \text{and} \qquad \text{at} \quad x = 1 \qquad u_x = -3u$$

and with intial condition at $t = 0, u = x(1 - x)$.
Using the following methods, write out the finite difference equations (FDE) to be used for the computer programming (use the variable $r = \alpha\Delta t/(\Delta x)^2$).
Write them in the following order:
FDE at left end
FDE in interior
FDE at the right end
Do this for
(a) Explicit method with one sided derivative used at x=1.
(b) Implicit method with false boundary used at x=1.
(c) Crank-Nicholson with false boundary used at x=1.