

1. Find the modified equation for Lax-Wendroff and Leapfrog scheme and compare the numerical properties of these schemes. Derive each modified equations to the term u_{xxxx} .
2. Write a computer code to solve linear first-order wave equation:

$$u_t + au_x = 0, \quad t > 0$$

$$u(x, 0) = \begin{cases} 1, & 0 < x \leq 10 \\ 0, & 10 < x \leq L \end{cases}$$

and

$$u(0, t) = 1$$

Find the exact solution. Then Solve the problem with these numerical schemes:

- (a) Lax method
- (b) Lax-Wendroff method
- (c) Mac-Cormack method

for $\Delta x = 1$ and $L = 50$ with 51 grid points. Adopt proper numerical boundary condition at $x = L$ if it is necessary for any of these methods.