**Chapter I: Introduction**
**Chapter II: Basic Concepts**

2.1 Introduction to Second-order PDE
2.2 Classification of Second-order PDE
2.3 Cauchy Problem and Cauchy-Kowalewsky Theorem
2.4 Well-posed and Ill-posed problems
2.5 Qualitative Properties of Elliptic, Parabolic and Hyperbolic PDE

**Chapter III: Finite-Difference Calculus**

3.1 Taylor Series Expansion
3.2 Order Symbol and Gauge Function
3.3 Truncation Error
3.4 Finite Difference Approximation to a Derivative
3.5 Higher-order approximation
3.6 Richardson's Extrapolation Method
3.7 Difference Operator Theory
3.8 Implicit Finite Difference Formula

**Chapter IV: Difference Method for Parabolic PDE's**

4.1 Explicit Finite-Difference Approximation to the Heat Equation
4.2 Concept of Convergence
Worked Example and Comparison with Analytical Solutions
4.3 Propagation Speed of Disturbance and its Relation to Stability
4.4 Implicit Finite-Difference Approximation to the Heat Equation
Thomas Algorithm
Crank-Nicalson Method
Keller-Box Method
Block-Tridiagonal System
Treatment of Derivative Boundary Condition
4.5 Fortran Hints in Programming
4.6 Roundoff Error and Worked Example
4.7 Diagonal Dominancy

**Chapter V: Stability of Finite Difference Method**

5.1 Exact Solution of Heat Eq. by Fourier Series
5.2 Exact Solution of the Finite-Difference Approximation of the Heat Equation
5.3 Definition of Stability
5.4 Von-Neumann Stability Analysis
5.5 Gerschgorin Circle Theorem
5.6 Matrix Stability Method
5.7 Examples

**Chapter VI: Finite-Difference Methods for Elliptic Equations**

6.1 Introduction
6.2 Finite-Difference Approximation to Poisson's Equation
6.3 Existance and Uniqueness of the Solution
6.4 Jocobi Iteration and its Rate of Convergence
6.5 Gauss-Seidal Iteration
6.6 SOR or Successive Over-Relaxation Method

**Chapter VII: Finite Volume Method (FVM)**

7.1 Governing Equations of Fluid Dynamics Used for FVM
7.2 The Finite Volume Method for Diffusion Problem
Worked Examples: One-Dimensional Steady State Problem
7.3 The Finite Volume Method for Convection-Diffusion Problem
Worked Examples: One-Dimensional Convection and Diffusion Problem
7.4 Properties of FVM
Conservativeness, Boundedness and Transportiveness
7.5 Solution Algorithms for Pressure-Velocity Coupling in Steady flow
Staggered Grid, SIMPLE, SIMPLEC and PISO Algorithms

**Chapter VIII: Hyperbolic Equations (Theory)**

8.1 Constant Coefficient Advection Equation (1-D Wave Eq.)
8.2 Initial-Value Problem
8.3 Initial-Boundary Value Problem
8.4 Characteristic Theory for Linear and Quasi-linear Wave Equations
Examples
8.5 Conservation Law Equation
8.6 Characteristic Intersection and Shock Formation
8.7 Weak or Generalized Solution
Rankine-Hugoniot Condition
Gas Dynamics Shock Wave
8.8 One-Dimensional Euler Equations (Primitive and Conservative Forms)
8.9 Mathematical Properties of Euler Equations
8.10 Eigenvalues and Compatibility Relations
8.11 Characteristic Variables
8.12 Propagation of Flow Quantities, Boundary Treatment

**Chapter IX: Numerical Solution of Hyperbolic Equations**

9.1 Method of Characteristics
Examples
9.2 Explict Finite Difference for 1-D Wave Equation
FTFS, FTBS, FTCS, Lax-Fridrichs Method, CTCS, Lax-Wendroff Methods
9.3 Von-Neumann Stability Analysis (Revisited)
Stability Analysis of above Methods
9.4 CFL Stability and its Physical Significance
9.5 Implicit Finite Difference Methods
9.6 Scalar Conservation Laws Formulation
9.7 Monton, Upwind and Downwind Methods
9.8 Discretization Methods for One-Dimensional Euler Equation

**TEXT:** Class Notes

**Grading:** The following weights will be used for grading (Total=100):

| | | | |
|---|---|---|---|
| Homeworks | 10% | First Midterm | 15% |
| Quizes | 25% | Second Midterm | 15% |
| Projects | 10% | Final Exam | 25% |