Homework #2	**Computational Fluid Dynamics I**	Prof. V. Esfahanian
Due Date: 15/7/97		TA: A.Bahrami & M.Fazli

1. Classify the following PDE's:

   a) $u_{xx} + 3u_{xy} + 2u_{yy} = 0$

   b) $x^2 u_{xx} + 2xyu_{xy} + y^2 u_{yy} + 3u_x = 0$

   c) $x^2 u_{xx} - 2xyu_{xy} + y^2 u_{yy} + 3u_x = 0$

   d) $(1 - x^2)u_{xx} - 2xyu_{xy} + (1 - y^2)u_{yy} = 0$

   e) $(1 - u^2)u_{xx} + u_{yy} = 0$

2. (a) Show that the one-dimensional Navier-Stokes equation without pressure gradient (known as the "viscous" Burger's equation) is parabolic in x, t.

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \alpha\frac{\partial^2 u}{\partial x^2}$$

   (b) Now, How about inviscid Burger's equation?

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$$

3. The Tricomi equation governs problems of the mixed type such as inviscid transonic flows. Classify this equation.

$$y\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

4. Show that the following system of equation is of an elliptic nature.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

5. Transform the wave equation,

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0$$

   into generalized coordinates $\xi = \xi(t, x)$, $\eta = \eta(t, x)$ and show that the resulting equation is hyperbolic.