

1. For  $u_t = \alpha u_{xx}$  the following discretization is proposed

$$\frac{3}{2} \frac{\Delta_t u_i^n}{\Delta t} - \frac{1}{2} \frac{\nabla_t u_i^n}{\Delta t} = \frac{\alpha}{(\Delta x)^2} \delta_x^2 u_i^{n+1}$$

- (a) write this out in suitable form for the computer using the variable  $r = \alpha \Delta t / (\Delta x)^2$ .  
(b) Is this method explicit, or implicit? How many time levels are involved?  
(c) Find the truncation error at the point  $(x_i, t_{n+1})$ .

2. Consider the following parabolic PDE:

$$u_t = \alpha u_{xx} - 2u \quad 0 < x < 1, \quad t > 0$$

with boundary conditions

$$\text{at } x = 0 \quad u = 1 \quad \text{and} \quad \text{at } x = 1 \quad u_x = -3u$$

and with initial condition at  $t = 0, u = x(1 - x)$ .

Using the following methods, write out the finite difference equations (FDE) to be used for the computer programming (use the variable  $r = \alpha \Delta t / (\Delta x)^2$ ).

Write them in the following order:

- FDE at left end
- FDE in interior
- FDE at the right end

Do this for

- (a) Explicit method with one sided derivative used at  $x=1$ .
- (b) Implicit method with false boundary used at  $x=1$ .
- (c) Crank-Nicholson with false boundary used at  $x=1$ .